



**General Certificate of Education (A-level)
January 2011**

Mathematics

MFP4

(Specification 6360)

Further Pure 4

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP4

Q	Solution	Marks	Total	Comments
1(a)	$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ x+y+z & y+z+x & z+x+y \end{vmatrix}$	M1		e.g. $R_3' = R_3 + R_2$
	$= (x+y+z) \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$	A1	2	
(b)	Expanding remaining det. $\Delta = (x+y+z)(x-2y+z)$	M1 A1	2	
Total			4	
2	$c = \mathbf{a} \times \mathbf{b} = ab \sin \theta$ $d = \mathbf{a} \cdot \mathbf{b} = ab \cos \theta $ $c^2 + d^2 = a^2 b^2 (\cos^2 \theta + \sin^2 \theta) = a^2 b^2$	B1 B1 B1	3	Condone lack of - Legitimately shown
Total			3	
3(a)	$\begin{vmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{vmatrix} = 8t^2 - 7t - 1 = 0$	M1 M1		Attempt at det. of coefft. mtx. (or equivalent) Equating to zero and solving a quadratic eqn. in t
	$t = 1, -\frac{1}{8}$	A1	3	
(b)	$\begin{aligned} x + 2y + 3z &= a \\ t = 1 \Rightarrow 2x + 3y - z &= b \\ 3x + 5y + 2z &= c \end{aligned}$	B1✓		FT any integer value found
	E.g. ① + ② - ③ $\Rightarrow a + b = c$	M1 A1	3	
Total			6	
4(a)	(i) $\mathbf{X}^2 = \begin{bmatrix} 23 & 1 & -3 \\ 2 & 24 & 3 \\ -4 & 2 & 19 \end{bmatrix}$ $\mathbf{X}^2 - \mathbf{X} = 20\mathbf{I}$ i.e. $k = 20$	M1 A1 A1	3	≥ 5 correct for the M All 9 correct for the A Shown legitimately
	(ii) Mult ^g . $\mathbf{X}^2 - \mathbf{X} = 20\mathbf{I}$ by \mathbf{X}^{-1} Re-arranging $\mathbf{X} - \mathbf{I} = 20\mathbf{X}^{-1}$ to get $\mathbf{X}^{-1} = \frac{1}{20}(\mathbf{X} - \mathbf{I})$	M1 A1	2	Legitimately
(b)	$\mathbf{X}^{-1} = \frac{1}{20} \begin{bmatrix} 2 & 1 & -3 \\ 2 & 3 & 3 \\ -4 & 2 & -2 \end{bmatrix}$	B1		Noted or used
	$(\mathbf{XY})^{-1} = \mathbf{Y}^{-1} \mathbf{X}^{-1}$	M1		Incl. attempt at the multn.
	$\begin{bmatrix} 6 & 3 & -9 \\ 2 & -1 & 1 \\ 2 & 3 & 3 \end{bmatrix}$	A1	3	
Total			8	

MFP4(cont)

Q	Solution	Marks	Total	Comments
5(a)	$I_1: 6x + 2y + 9z = 5$ $I_2: 10x - y - 11z = 4$	B1	1	Both
(b)	<p>Method 1</p> <p>E.g. $I_1 + 2 I_2 \Rightarrow 13(2x - z = 1)$</p> $\frac{x}{1} = \frac{z+1}{2} = \lambda$ <p>Using 1st two to find 3rd in terms of λ: $x = \lambda, z = 2\lambda - 1 \Rightarrow y = 7 - 12\lambda$</p> $\mathbf{r} = \begin{bmatrix} 0 \\ 7 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$ <p>Method 2</p> <p>$(0, 7, -1)$ or $(\frac{7}{12}, 0, \frac{1}{6})$ or $(\frac{1}{2}, 1, 0)$</p> $\begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} \times \begin{bmatrix} 10 \\ -1 \\ -11 \end{bmatrix} = (\pm)13 \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$ $\mathbf{r} = \begin{bmatrix} 0 \\ 7 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$	M1 M1 A1 M1 A1	5	Parametrisation attempt Any correct vector eqn.form
(c)	<p>Subst^e. $x = \lambda, y = 7 - 12\lambda, z = 2\lambda - 1$ into $5x + 3y + 11z = 28$ Solving a linear eqn. in λ:</p> $\lambda = -2$ <p>$(-2, 31, -5)$</p>	M1 M1 A1 B1✓	4	CAO FT their λ in their line eqn.
(d)	$\mathbf{n} = \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix}$ $d = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix} = 10$ $\mathbf{r} \cdot \begin{bmatrix} 1 \\ -12 \\ 2 \end{bmatrix} = 10$ <p>Alt. $\mathbf{r} = \begin{bmatrix} 4 \\ 1 \\ 9 \end{bmatrix} + \lambda \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} + \begin{bmatrix} 10 \\ -1 \\ -11 \end{bmatrix}$</p>	B1✓ M1 A1✓	3	FT when used as part of a plane eqn., <i>not</i> a line Attempted Any correct vector eqn. form FT
Total			13	Plane eqn. attempt Point + at least 1 d.v. All correct

MFP4(cont)

Q	Solution	Marks	Total	Comments
6(a)	$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 15 \\ 16 \end{bmatrix} = 12 + 15 - 16 = 11 \text{ shown}$	B1	1	
(b) (i)	eqn., or use, of line incorporating p.v. $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and d.v. $\begin{bmatrix} 12 \\ 15 \\ 16 \end{bmatrix}$ $\frac{x-1}{12} = \frac{y-1}{15} = \frac{z+1}{16}$	M1 A1	2	
(ii)	$\sqrt{12^2 + 15^2 + 16^2} = 25$	B1✓		FT
	$\frac{12}{25}, \frac{15}{25}, \frac{16}{25}$	B1✓	2	FT
(c)	Use of $\mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 12 \\ 15 \\ 16 \end{bmatrix}$ with $\lambda = \pm 2$ (25, 31, 31) and (-23, -29, -33)	M1 A1 A1	3	
(d)	Method 1 $PQ = 5$ Area $\Delta PMN = \frac{1}{2} \times PQ \times MN = 250$	B1 M1 A1	3	(Since $Q = \text{midpt. } MN$)
	Method 2 $\overrightarrow{PM} = \begin{bmatrix} 20 \\ 30 \\ 35 \end{bmatrix}, \overrightarrow{PN} = \begin{bmatrix} -28 \\ -30 \\ -29 \end{bmatrix}$	(M1)		Attempted
	$\overrightarrow{PM} \times \overrightarrow{PN} = (\pm)20 \begin{bmatrix} 9 \\ -20 \\ 12 \end{bmatrix} \text{ attempted}$	(M1)		gm. or Δ
	within an area formula Area $\Delta PMN = 250$	(A1)		CAO
	Total		11	

MFP4(cont)

Q	Solution	Marks	Total	Comments
7(a)	Attempt at Char.Eqn. $\lambda^3 - 9\lambda^2 + 24\lambda - 16 = 0$ Attempt at (at least) one linear factor $(\lambda - 1)(\lambda - 4)^2 = 0$ $\lambda = 1, 4, 4$	M1 A3,2,1 M1 A1 A1	7	One each following coefft.
(b)	$2x - y + z = 0$ $\lambda = 1 \Rightarrow -x + 2y + z = 0 \Rightarrow \alpha \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ $x + y + 2z = 0$ $\lambda = 4 \Rightarrow x + y - z = 0$ Choosing any two independent vectors satisfying this E.g.s: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	M1 A1 B1 M1 A1	5	Subst ^g . back and solving
(c)	$\alpha \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \equiv$ a line of invariant points e.g. $\alpha \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \equiv$ an invariant plane	M1 A1 B1	3	Invariant line LoIPs
Total			15	

MFP4(cont)

Q	Solution	Marks	Total	Comments
8(a)	$\text{Det}(\mathbf{M}) = -1$ Magnitude = 1 \Rightarrow area invariant -ve sign \Rightarrow cyclic order of vertices is reversed OR “reflection” involved	B1 B1✓ B1	3	FT area s.f.
(b)	Method 1 Char. Eqn.: $\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$ Subst ^g . back: $\lambda = 1 \Rightarrow y = \frac{1}{2}x$ and $\lambda = -1 \Rightarrow y = \frac{1}{4}x$	M1 A1 M1 A1 A1	5	Finding and solving attempt
	Method 2 $\begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ mx \end{bmatrix} = \begin{bmatrix} (8m-3)x \\ (3m-1)x \end{bmatrix}$ Use of $y' = mx'$: $3m - 1 = 8m^2 - 3m$ Solving a quadratic eqn. in $m = \frac{1}{4}, \frac{1}{2}$ $p = \frac{1}{2}$ and $q = \frac{1}{4}$	(M1) (M1) (M1A1) (A1)		Attempted From $(4m - 1)(2m - 1) = 0$
(c)	(i) $p = \frac{1}{2} = \tan\theta$ $\Rightarrow \cos 2\theta = \frac{3}{5}$ and $\sin 2\theta = \frac{4}{5}$ $\mathbf{R} = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$	M1 A1	2	For these attempted and used in a reflection matrix
	(ii) Use $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix} \mathbf{S} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$ \mathbf{S} found using inverse matrix $= \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix} \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -13 & 36 \\ -9 & 23 \end{bmatrix}$ Shear, parallel to $y = \frac{1}{2}x$ mapping (e.g.) $(1, 1) \rightarrow (4.6, 2.8)$	M1 M1 A1 B1 B1✓	5	FT their \mathbf{R} Or equivalent method CAO FT any pt. and its image
	Total		15	
	TOTAL		75	